Features for Heartbeat Sound Signal Normal and Pathological

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Abstract: This paper is concerned with a synthesis study of the fast Fourier transform (FFT) and the wavelet transform in analysing the phonocardiogram signal (PCG). It bas been shown that the wavelet transform provides enough features of the PCG signals that will help clinics to obtain qualitative and quantitative measurements of the time-frequency PCG signal characteristics and consequently aid in diagnosis. Similarly, it bas been shown that the frequency content of such a signal can be determined by the FFT without difficulties. Abnormal heartbeat sounds may contain, in addition to the first and second sounds, S1 and S2, murmurs and aberrations caused by different pathological conditions of the cardiovascular system.

Keywords: Heart auscultation, spectral analysis, time-frequency representation, component localisation, time delay, error of reconstruction, classification.

I. INTRODUCTION

Heartbeat sound analysis by auscultation is still insufficient to diagnose some heart diseases. It does not enable the analyst to obtain both qualitative and quantitative characteristics of the phonocardiogram signals [1, 2].

Abnormal heartbeat sounds may contain, in addition to the first and second sounds, S1 and S2, murmurs and aberrations caused by different pathological conditions of the cardiovascular system [2]. Moreover, in studying the physical characteristics of heart sounds and human hearing, it is seen that the human ear is poorly suited for cardiac auscultation [3]. Therefore, clinic capabilities to diagnose heart sounds are limited.

The sound emitted by a human heart during a single cardiac cycle consists of two dominant events known as the first heart sound S1 and the second heart sound S2. S1 relates to the closing of the mitral and tricuspid valves whilst S2 is generated by the halting of the aortic and pulmonary valves leaflets [1]. The sound S1 corresponds in timing to the QRS complex in ECG (Electrocardiogram) and the sound S2 follows the systolic pause in the normal cardiac cycle (Fig. **1**).

The characteristics of the PCG signal and other features such as heart sounds S1 and S2 location; the number of components for each sound; their frequency content; and their time interval (or split S2), all can be measured more accurately by digital signal processing techniques.

The FFT (Fast Fourier Transform) can provide a basic understanding of the frequency contents of the heart sounds. However, FFT analysis remains of limited values if the stationary assumption of the signal is violated. Since heart sounds exhibit marked changes with time and frequency, they are therefore classified as non - stationary signals. To understand the exact feature of such signals, it is thus important, to study their time - frequency characteristics.

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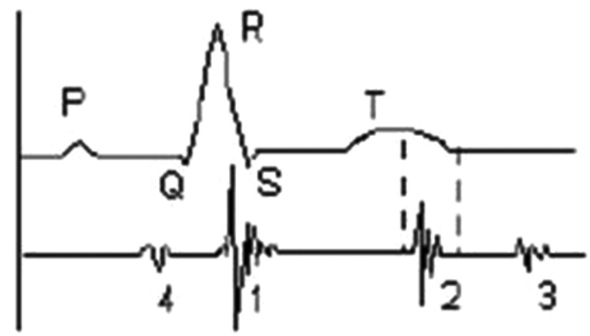

Fig. (1). Relationship between the heart sounds and the ECG waveform.

In this paper, the wavelet transform, continue version (CWT) or discrete (DWT) are used to analyse both the normal and abnormal heart sounds in both time and frequency domains. This technique has been shown to have a very good time resolution for high-frequency components. In fact, the time resolution increases as the frequency increases and the frequency resolution increases as the frequency decreases [4, 5].

Furthermore, the wavelet transform has demonstrated the ability to analyse the heart sound more accurately than other techniques STFT or Wigner distribution [6] in some pathological cases.

However, the Wigner distribution (WD) and the corresponding WVD (Wigner Ville Distribution) have shown good performances in the analysis of non-stationary signals. This comes from the ability of the WD to separate signals in both time and frequency directions. One advantage of the WD over the STFT is that it does not suffer from the time-frequency trade-off problem. On the other hand, the WD has a disadvantage since it shows cross-terms in its response. These cross-terms are due to the nonlinear behaviour of the WD and bear no physical meaning. One way to remove these cross-terms is by smoothing the time-frequency plane, but this will be at the expense of decreased resolution in both time and frequency [7, 8]. The wavelet Transform is a technique in the domain of time-frequency distributions. The main idea of this method is the representation of an arbitrary

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signal as a superposition of basic signals, "atoms", located in time and frequency. These atoms may be derived by means of a special operation on a single parent atom. Parent atoms and derivation operation are usually chosen such as to enable the construction of an orthonormal system [9].

In this paper, the FFT and the WT methods are applied to analyse normal and pathological PCG signals. Therefore, in this paper we are interested in analysing the second heart sound S2, which consists of two major components, one due to the closure of the aortic valve (A2) and the other due to the closure of the pulmonary valve (P2). The aortic valves normally close before the pulmonary valves, leading to a time delay between these two components A2 and P2. This delay is known as the "split" in medical community [10-12]. The importance of S2 in diagnosis has long been recognised and its significance is considered by cardiologist as the "key" to auscultation of the heart [13].

Specifically during expiration, A2 and P2 are separated by a relatively short interval typically less than 30ms [14]. A2 has higher frequency contents than that of P2 and generally A2 precedes P2. It is therefore required to apply a signal processing tool to analyse the PCG and consequently measure these time intervals and frequency content of S2.

II. THEORETICAL BACKGROUND

II.1. Fast Fourier Transform (FFT)

The Fourier transform S(w) of a signal s(t) is defined as :

$$S(w) = \int s(t).\exp(-jwt)dt \quad (1)$$

Where t and w are the time and frequency parameters, respectively. S(w) defines the spectrum of the signal s(t). It consists of components at all frequencies over the range for which s(t) is non zero. Methods and systems for long term monitoring of one or more physiological parameters such as respiration, heart rate, body temperature electrical hear activity, blood oxygenation, blood flow velocity, blood pressure, intracranial pressure, the presence of emboli in the blood stream and electrical brain activity are provided [15].

II.2. Wavelet Transform

Wavelet transforms have become well known as useful tools for various signal processing applications. The continuous wavelet transform is best suited to signal analysis [16]. Its semi-discrete version (wavelet series WS) and its fully discrete one (the discrete wavelet transform DWT) have been used for signal coding applications, including image compression [17] and various tasks in computer vision [18].

Given a time-varying signal s (t), wavelet transforms consist of computing coefficients that are inner products of the signal and a family of "wavelets". In a continuous wavelet transforms, the wavelet corresponding to scale "a" and time location "b" is:

$$\Psi(a,b) = \frac{1}{\sqrt{/a/}}\Psi(\frac{t-b}{a}) \quad (2)$$

Where Ψ(t) is the "mother wavelet", which can be thought of as a band-pass function. The factor $\sqrt{/a/}$ is used to ensure energy preservation [16]. There are various ways of discretising time-scale parameters (b,a), each one yielding a different type of wavelet transform.

The continuous wavelet transform (CWT) was originally introduced by G.Grossmann and J.Morlet [19]. Time t and the time-scale parameters vary continuously.

$$CWT\{s(t);a,b\} = \int s(t)\,\Psi(a,b)^*(t)dt \quad (3)$$

(The asterisk stands for complex conjugate).

Wavelet series (WS) coefficients are sampled CWT coefficients. Time remains continuous but time-scale parameters (b,a) are sampled on a so-called "dyadic" grid in the time-scale plane (b,a) [20]. A common definition is:

$$C_{jk} = CWT\{s(t);\ a = 2^{j},\ b = k2^{j}\} \quad j,k \in Z \quad (4)$$

The wavelets are in this case:

$$\Psi_{jk}(t) = 2^{-j/2}\quad \Psi(2^{-j}t - k) \quad (5)$$

Wavelet series have been popularised under the form of signal decomposition onto "orthogonal wavelets" by Meyer, Mallat, Daubechies and others authors.

The discrete wavelet transform (DWT) has been recognised as a natural wavelet transform for discrete-time signals. Both the time and time-scale parameters are discrete.

The discretisation process partially depends upon the algorithm chosen to perform the transformation.

The $C_{j,k}$ could be well approximated by digital filter banks, by using Mallat's [21] remarkable fast pyramid algorithms, which involve use of low-pass and high-pass filters (Fig. **2**).

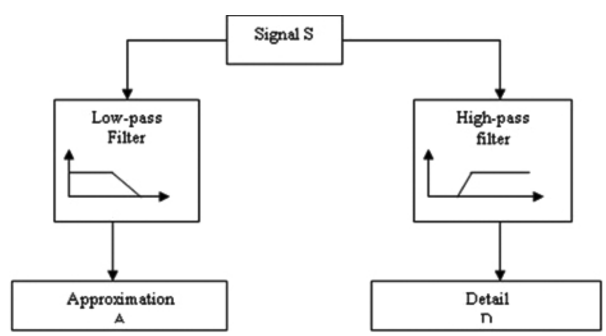


Fig. (2). Decomposition an approximation and detail of the signal S.

The Mallat algorithm is in fact a classical scheme known in the signal processing community as two-channel subband coder.

This very practical filtering algorithm yields as fast wavelet transform (WFT). For many signals, the low-frequency content is the most important part. It is what gives the signal its identity. The high-frequency content, on the other hand, imparts flavour or nuance.

For example, if we consider the human voice, if the high-frequency component is removed, the voice sounds different, but you can still get what is being said. However, if you remove enough of the low-frequency components, you hear gibberish. It is for this reason that, in wavelet analysis, we often speak of approximation and details. The invention relates to the method of cardio-acoustic signal analysis which comprises the steps of receiving a signal representative of heart sounds from an acoustic sensor, representing the received signal in a time-frequency-amplitude domain, where the method further comprises the step of applying a psycho-acoustic model to the received signal in the time-frequency amplitude domain in order to generate a representation of the signal in a time - perceptual frequency - perceptual loudness domain [22].

The approximation is the high scale, low-frequency components of the signal. The details are the low scale, high-frequency components. The filtering process, at its most basic levels, looks like this.

The original signal S passes through two complementary filters and emerges as two signals: signal approximation "A" and signal detail "D" Fig. (**2**).

The decomposition process can be iterated, with successive approximations being decomposed in turn, so that one signal is broken down into many lower-resolution components. This is called the "wavelet decomposition tree" Fig. (**3**).

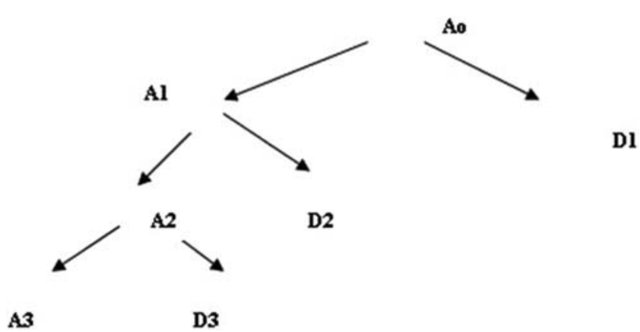


Fig. (3). Wavelet decomposition tree of one example of 3 levels.

III. RESULTS AND DISCUSSION

The Fast Fourier Transform (FFT) and the Continuous Wavelet Transform (CWT) techniques are applied to analyse different PCG signals. In fact, four cases are considered, one normal and three abnormal (or pathological). The sampling rate used is 8000 samples/s. The scale of both time and frequency axis is a linear scale. The frequency scan is from 1Hz to 500Hz.

III.1. Frequency Analysis of the PCG (FFT)

An FFT algorithm is first applied on the PCG signal given in Fig. (**4a**). This figure shows a normal cardiac (heartbeat sound) cycle where the two major sounds S1 and S2 are clearly depicted. The frequency spectrum illustrated in Fig. (**4a**) shows that the normal PCG signal has a frequency content varying from around 40Hz up to 200Hz. At this stage, the sound S1 or S2 cannot be separated. In fact, the application of the FFT on heart sounds S1 and S2 after their separation or identification [12, 13] shows that the basic frequency components are obviously detected by the Fourier transform but not the time delay between these components. The two components A2 (due to the closure of the aortic valve) and P2 (due to the closure of the pulmonary valve) of the second sound S2 are obvious in Fig. (**1c**). However, the FFT analysis of S2 cannot tell which of two, either A2 or P2 precedes the other or the value of the time delay between them. For a normal heart activity, usually A2 precedes P2 and the value of time delay between them is lower than 40ms [9, 10]. These parameters (position and time delay) of A2 and P2 are very important to detect some pathological cases.

The sound S2 seems to have higher frequency content than that of S1 as shown in Fig. (**4b**) and Fig. (**4c**). The spectrum of S1 has reasonable values in the range 10-200Hz. The spectrum of the sound S2 has reasonable values in the range 50-300Hz. The spectrum for this sound is distinctly resolved in time into two major's components (A2 and P2) as shown in Fig. (**4c**).

III.2. Continuous Wavelet Analysis of the PCG

An algorithm of the Continuous Wavelet Transform is applied to analyse the PCG signal of a normal cardiac cycle illustrated in Fig. (**1c**). Fig. (**3a**) shows the result of this analysis. The two heart sounds are clearly shown in dark colour. There is space of 2500 samples corresponding to 0.312 seconds. A system, method and computer executable code for likely generating of cardiovascular disease from acquired cardiovascular sound signals is disclosed. Also disclosed is a system, method, and computer executable code for collecting, forwarding and analysing cardiovascular sound signals, where the collecting and analysing may occur at locations that are remote from each other. This study further discloses a system, method and computer executable code for determining the time and phase information contained in cardiovascular sound signals [23].

The continuous wavelet transforms of S1 and S2 are also displayed separately in Fig. (**6b**) and Fig. (**6c**), respectively. As it is illustrated in Fig. (**6c**), the sound S2 is shown to have higher frequency content than that of the S1. This is expected since the amount of blood present in the cardiac chambers is smaller [2, 14].

Besides this, the spectrum of S1 is clearly resolved in time into four major components [15]. The spectrum of the sound S2 is resolved in time into two major's components (A2 and P2). These results confirmed those found by spectral analysis (FFT technique), see Fig. (**4b**) and (**4c**). The time delay between A2 and P2 can be easily measured with the use of the wavelet coefficients see Fig. (**5c**). This delay is measured to be 13ms. It is smaller than the 30ms as seen in the normal pathological conditions of the PCG signal [24].

Pathological conditions could cause this time difference to narrow or widen. Moreover, the order of occurrence of A2 and P2 may be reversed. The wavelet transform allows measurement and determination of this time difference and thus allows a diagnostic process regarding this important parameter to be produced. Moreover, the ability of the wavelet transform in heart disease diagnosis can be studied by

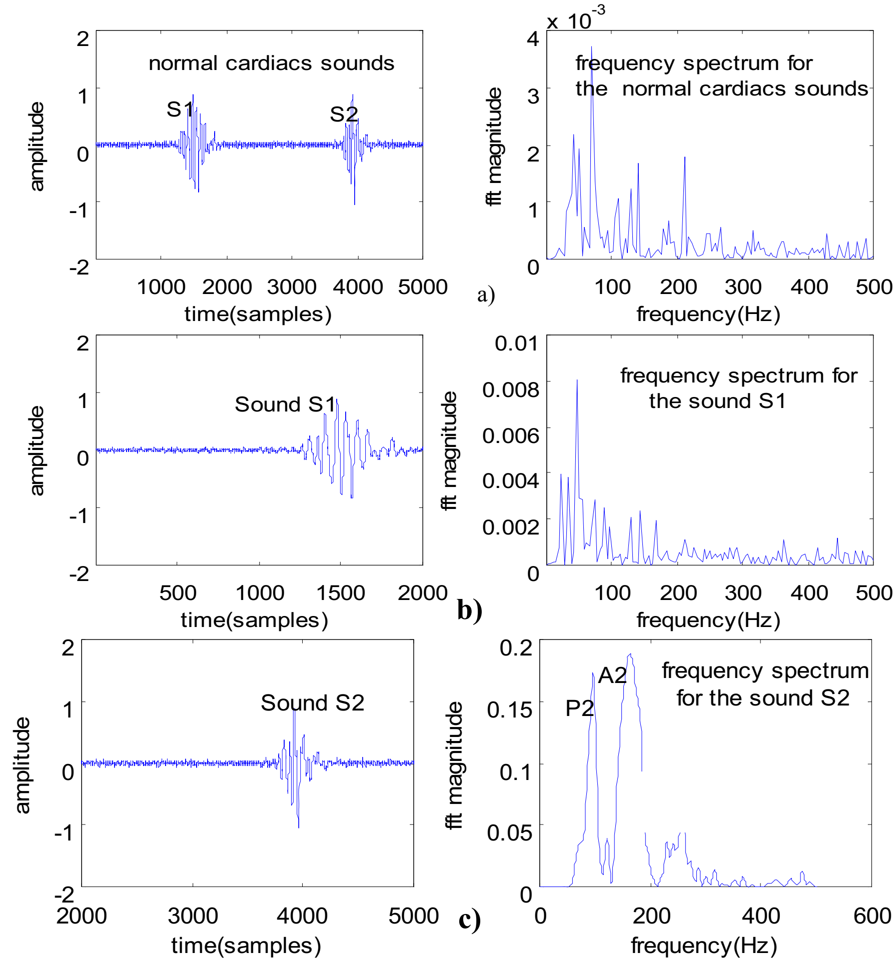
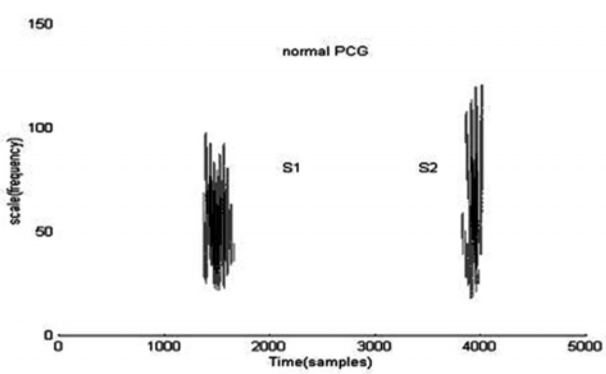


Fig. (4). Frequency spectrum for the normal cardiac's sounds and the sounds S1 and S2.

applying the CWT algorithm on different pathological cases. The results of this application are illustrated in Fig. (**6a**) (aortic-insufficiency), Fig. (**6b**) (aortic-stenosis) and Fig. (**6c**) (mitral-stenosis). The coefficients of the CWT allow us to clearly discern the frequency range of each signal. It also shows the major components according to the temporary variation; the maximal amplitude is characterised by a darker colour than that of the small amplitudes.

III.3. Identification and Measure of the Split S2

The second heart sound S2 consists of two acoustic components A2 and P2, the former is due to the closure of the aortic valve and the latter is due to the closure of the pulmonary valve. The aortic valve usually closes before the pulmonary valve, introducing a time delay known as "split". A



(Fig. 5. Contd….)

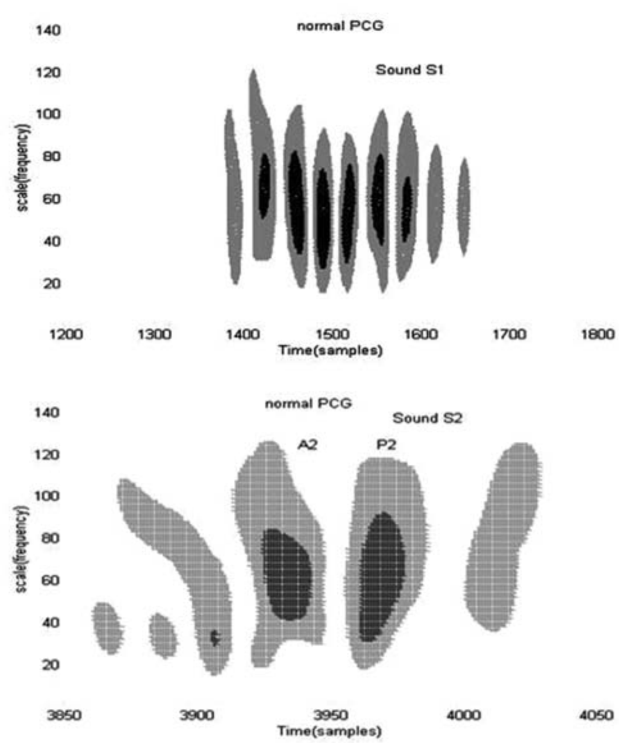


Fig. (5). Coefficients of the continuous wavelet transorm (CWT) for the normal PCG of: a) Oe cycle b) sound S1 c) sound S2.

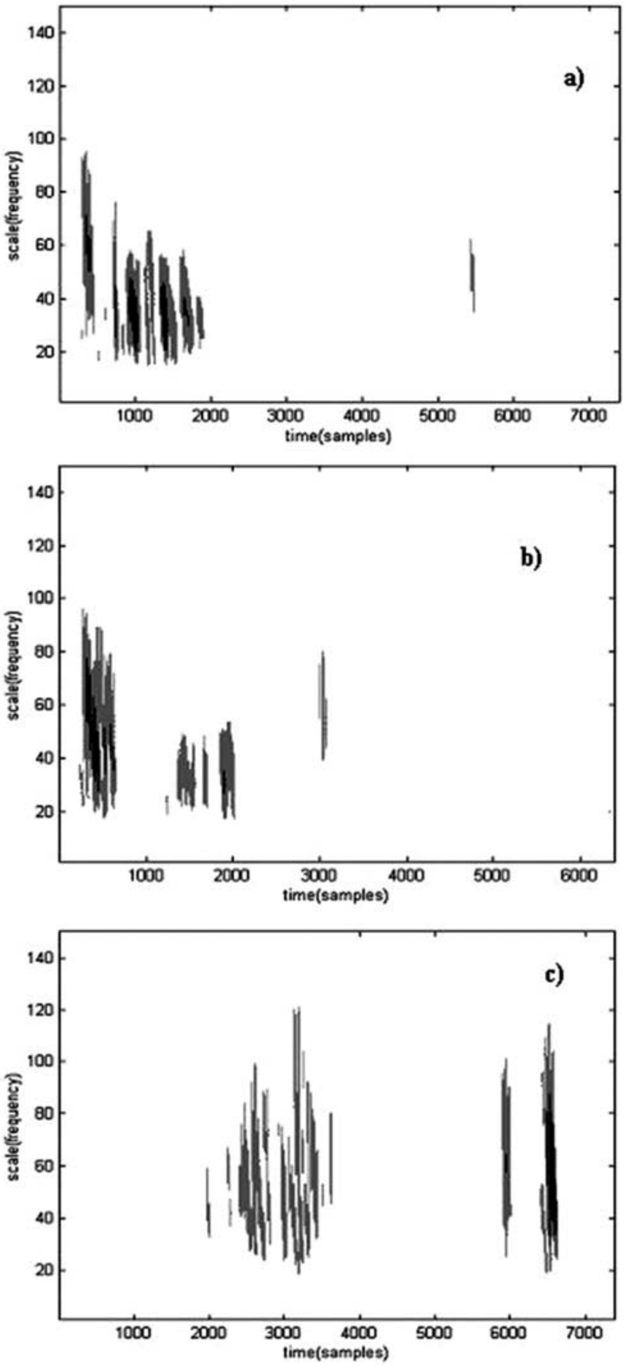


Fig. (6). Coefficients of the continuous wavelet for the abnormals cardiac sounds: a) aortic-coarctation b) aortic-stenosis c) mitral-stenosis.

technique based on the discrete wavelet transform (DWT) and the continuous wavelet transform (CWT) is developed in this paper to measure the split, to quantify the splitting, two components in S2 (i.e. A2 and P2) are identified and the delay between the two components can be estimated. One normal case and three pathological cases (mitral stenosis ,pulmonary stenosis and atrial septal defect) are considered in this study. For each sound S2 of the considered signals,

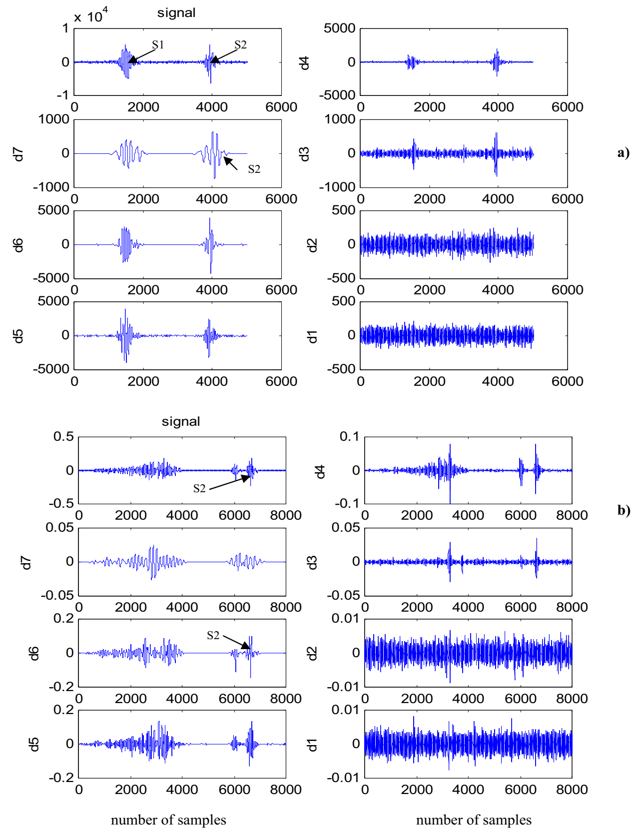


Fig. (7). wavelet decomposition of a) the normal case (N) b) mitral stenosis (MS).

the split is measured. The main constitution of this paper is in the identification and the measurement of the split of the sound S2. The study confirms the notion of "variable splitting" for the normal phonocardiogram and "fixed splitting" for the ASD and the PS cases.

In this work, we limit our study to recognising, measuring and identifying the split S2 of the normal and pathological heartbeats. Our approach has been developed on the basis such as:

a) The use of the discrete wavelet transform (DWT) as a filtering technique in order to locate and identify the best split S2.

b) The result of stage a) is then used by the continuous wavelet transform (CWT) in order to measure the split S2.

Our study is concerned with PCG signals, one normal and three pathological. Table **1** specifies then various signals. The signals PCG were normalised in energy to take into account the disparity in magnitude due to the different amplification used during acquisition as well as the variation induced by the lead sites. The decomposition (multi resolution analysis) of the PCG signals by the DWT depicts different energy partitions for resolution levels of the beats under study. The high frequency information is localized on the coarser levels whereas the split S2 generally appears on the

Table 1. Order and Best Levels Used for the Analysed Signals PCG

Type of signals	Abbreviation	Order of wavelet	Best level
Normal	N	db5	d7
Pulmonary stenosis	PS	db7	d6
Atrial septal defect	ASD	db10	d3
Mitral stenosis	MS	db7	d6

finest scales. This result is illustrated in Figs. (**8-9**). Consequently, based on different tests and our experience, the number of decomposition levels and the order of the wavelet used are given in Table **1**. The wavelet “db”(Daubechies) is used, since it has oscillations very similar to those of signals PCG analysed. This involves a method and apparatus for analysing measured signals, including the determination of a measurement of correlation in the measured signals during a calculation of a physiological parameter of a monitored patient. Use of this invention is described in particular detail with respect to blood oxymetry measurements [25].

Table **1** provides also the levels of the details allowing a suitable measurement of the split S2 for the various signals.

The multi resolution analysis based on the discrete wavelet transform (DWT) is a powerful tool in filtering, separating and identification of the internal components and murmurs of the various analysed signals. The split is best depicted on level d7 for the normal phonocardiogram Fig. (**7a**) and on level d6 for the mitral stenosis Fig. (**7b**), on level d6 for the pulmonary stenosis Fig. (**8a**) and on level d3 and d4 for the atrial septal defect Fig. (**8b**).

From the different measurements which we carried and resumed in Table **2**.

III.4. Detection of Differences of the Phonocardiogram Signals: Temporal and Frequency Rates

In order to better quantify the differences existing between the sounds S1 and S2 of these signals, we will consider the temporal and frequency rates shown as follows:

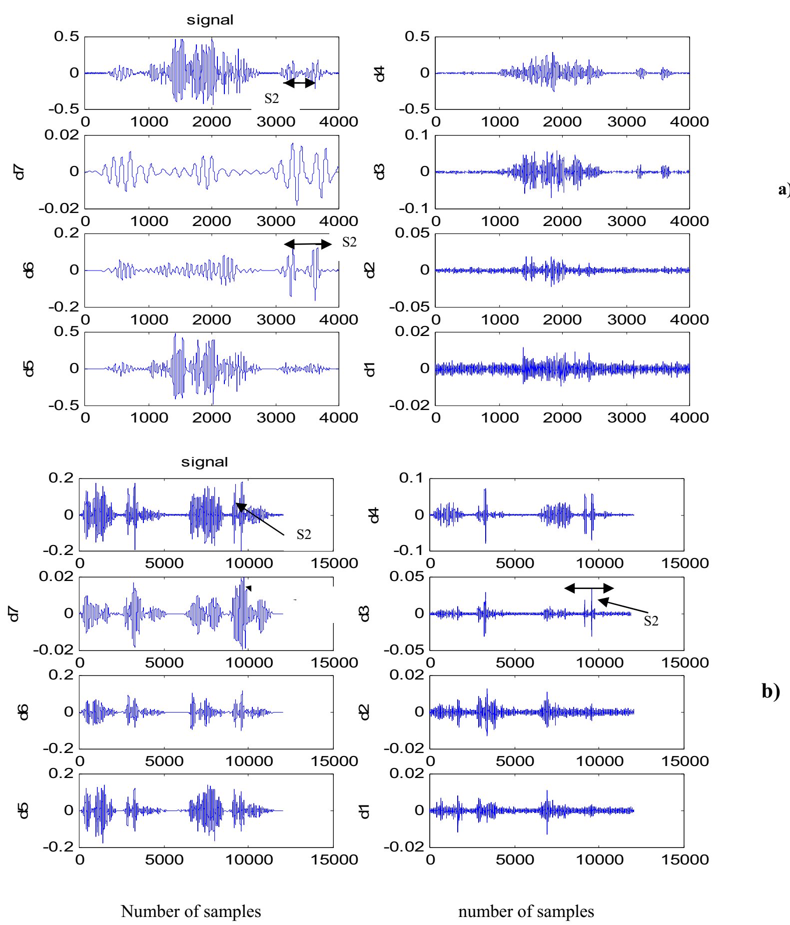


Fig. (8). Wavelet decomposition of a) pulmonary stenosis (PS) b) atrial septal defect (ASD).

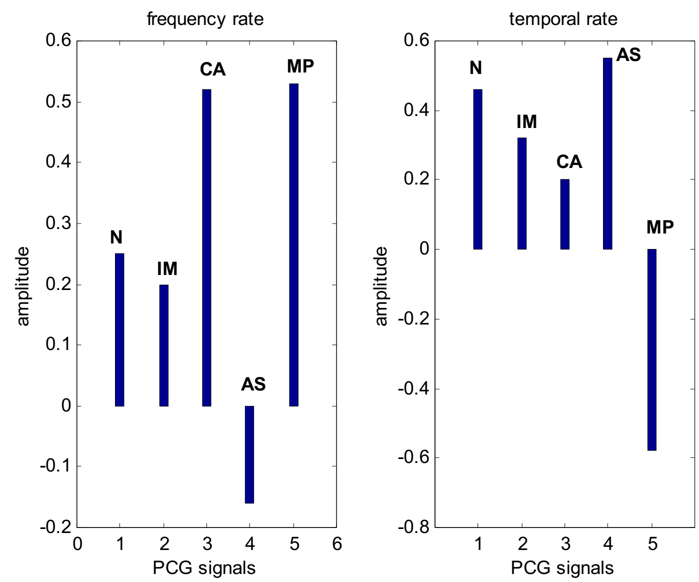


Fig. (9). Values of the frequency and temporal rate of the used PCG signals.

$$R_f = \frac{\Delta f(S2) - \Delta f(S1)}{\Delta f(S2)} \text{ : Frequency rate} \qquad (6)$$

and

$$R_t = \frac{\Delta t(S1) - \Delta t(S2)}{\Delta t(S1)} \text{ : Temporal rate} \qquad (7)$$

with :

Δf The frequency extend of the sound S1 or S2

and Δt : the temporal extend of the sound s1 or S2.

Table **3** provides PCG signals used in this analysis and Table **4** depicts the obtained results.

One easily notices according to values R_f and R_t provided in Table **4** and Fig. **9** that the values of the rates concerning the "IM" approach are greater than those of the case "CA ". For the case "AS", it is the significant variation of the frequency contents of the signal, which is highlighted by the negative sign of the frequency rate R_f.

For the case "MP ", it is the significant variation of the temporal contents of the signal, which is highlighted by the negative sign of the temporal rate R_f. This could be possibly explained by the significant variation of the temporal delay (or Split) between the two majors components of the second cardiac sound S2, aortic (A2) and pulmonary (P2), [15, 16].

CURRENT & FUTURE DEVELOPMENTS

This paper focuses on the development of the processing signal tools of the phonocardiogram signals (PCG).

To this end, one is constrained to reflect on the development and evaluation of methods of filtering and analysis in order to detect the various components constituting signal PCG. The parameters likely to contribute to the comprehension of this signal are also considered in this work.

Table 3. Used PCG Signals

PCG signals	Abbreviation
*** PCG signals without murmurs**	
Normal heartbeat cardiac sounds	N
Innocent murmur	IM
Coarctation of the aorta	CA
*** PCG signals with murmurs**	
aortic stenosis	AS
mitral prolapse	MP

The object of this research is to lead to an approach, which possibly answers the nature of the PCG signal and consequently allows the detection and presentation of the various components and parameters of this signal in order to develop powerful methods of classification, which may assist the doctors in decision-making in their diagnosis.

CONCLUSION

The cardiac (heartbeat sound) cycle of phonocardiogram (PCG) is characterised by transients and fast changes in frequency as time progresses. It was shown that basic frequency

Table 2. Measure of the sp Lit of Various Analyzed Signals

Type of signals	N	MS	PS	ASD
Split(ms)	6	4	38	43

Table 4. Values of the Frequency and Temporal Rate Concerning the Used Cardiac Signals

	R_f	R_t
N	0.25	0.46
IM	0.20	0.32
CA	0.52	0.20
AS	- 0.16	0.55
MP	0.53	- 0.58

content of PCG signal can be easily provided using FFT technique. However, time duration and transient variation cannot be resolved; the CWT wavelet transform therefore is a suitable technique to analyse such a signal. It was also shown that the coefficients of the continuous wavelet transform give a graphic representation that provides a quantitative analysis simultaneously in time and frequency. It is therefore very helpful in extracting clinically useful information.

It is also shown that the wavelet transform provides a detailed description of the structure of the cardiovascular sound cycle and provides a method with which analysis of heart sounds can be performed using a quantitative procedure based on timing, frequency, intensity, evolution and shape.

The discrete wavelet transform with the multi resolution analysis, is easy to decompose the signal into elementary building blocks and localise the best split S2 between the components A2 and P2. The continuous wavelet transform (CWT) after the localization of the split S2 gives a graphic representation that provides a quantitative analysis simultaneously in time and in frequency. It is therefore very helpful in extracting clinically useful information.

The emphasis of this paper is on the identification and the measurement of the split of the sound S2, which aims to take splitting into account. This split is shown to be correlated to frequency content variations and changes along with signal cycles. The results obtained from the different cases studied demonstrate that the split between the components A2 and P2 and the frequency content variation along with the split are predominant features in discrimination between pathological cases.

Therefore, CWT analysis can be successfully used to determine this split and frequency content variation and consequently aid in diagnosis.

The temporal and frequency rates suggested in this paper seem to be very useful parameters for discrimination between various PCG signals.

Since these differences are statistically asserted, the diagnosis of this class of pathology, as well as of other classes, may be made easier by the use of this process.

Results clearly demonstrate that the WT of recorded heart sounds can be used as a diagnostic tool in the evaluation for abnormalities on the aortic and pulmonary valves.

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